

Name:

Fall 2021 Stat 311 Exam 1

Instructions: Submit as a single PDF file titled P Last Name First Name Exam 1 (e.g. P Etkin Arkady Exam 1). Don't forget the P in front or you will turn into a goat! Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK! You are not presenting a valid Kafka Protocol at your peril.

1. One of my cats, Shpuntik (Шпунтик), is a fussy eater. Give him the same dish more than once in an interval of 3 days and he won't eat it! If I have an assortment of 5 dish types (10 cans each), how many menus can I make for the next 10 days? Assume that Shpuntik is getting one can a day. [10 pts]

2. Vasya, Dima, and Petya are strangers by fortune and not by spirit; Each randomly walks to one of 3 bars on Brighton Beach. If you must know, the bars are A (Алый Парус), B (Ботаник), and C (Сестра Милосердия). As naturalized citizens, the fellows are each equally likely to order one bottle of Vodka or one bottle of Whiskey. If one of them walked into bar A and ordered Vodka, what is the probability they will all meet in the same bar? [10 pts]

3. A deck of 52 cards is randomly shuffled and the cards are turned over one by one until the first ace appears. What is the probability that no King, Queen, or Jack appear before the first ace? [10 pts]
4. An inspector checks 9 identically looking transistors for defects. He finds that 3 transistors are malfunctioning and sets them aside for disposal. Unfortunately his apprentice mixes them back with the rest by mistake. What is the probability the inspector will have to test 6 or more transistors before locating the 3 broken ones among the 9? [10 pts]

5. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Calculate the number of blue balls in the second urn. [10 pts]
6. Julius Caesar is rumored to have said "Et tu, Brute?" with his last dying breath. If each breath contains about 10^{22} molecules and these molecules of last breath are thoroughly mixed among the 10^{44} molecules in the atmosphere, estimate the probability that you are inhaling at least one of these historical molecules at this moment. You may assume for simplicity that the molecules in your breath are inhaled (sampled) sequentially with replacement. [10 pts]

7. A fair coin is flipped until either HH or TH appears for the first time. Is one pattern more likely than another? Compute the probabilities. [10 pts]
8. k balls are randomly and sequentially drawn from a bowl containing n balls numbered ball #1 - ball # n . If the drawing is done with replacement, compute the probability of obtaining a monotone, non-decreasing sequence. E. g. 1, 2, 2, 2, 4, 5, 5, ... [10 pts]

9. An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{5}{12}$, respectively. Determine the probability that a randomly chosen employee will choose no supplementary coverage. [10 pts]
10. 5 chess players will each play one match against the other. How many outcomes are possible if each outcome is to record the result of every match? Each match can result in a win for one player or else it is a draw. [10 pts]

Extra Credit

11. Use a story (i.e. a combinatorial argument) to simplify

$$\binom{n-1}{i-1} \binom{n-1}{k} \binom{n-1}{j} + \binom{n-1}{i} \binom{n-1}{k-1} \binom{n-1}{j} + \binom{n-1}{i} \binom{n-1}{k} \binom{n-1}{j-1} \text{ where } i, j, k \geq 1 \text{ and } i + j + k = n$$

[10 pts]

12. Simplify the product $(2n - 1) \cdot (2n - 3) \cdot (2n - 5) \cdots 3 \cdot 1$ by realizing what is being counted and using a different counting procedure. In short, come up with a combinatorial argument. (Hint: How many ways are there for $2n$ students to break into groups of two?) [10 pts]